PG – 367

II Semester M.Sc. Examination, June/July 2015
(CBCS)
MATHEMATICS
208 SC : Mathematical Modelling and Numerical Analysis – I

Time : 3 Hours Max. Marks : 70

Instructions : *Answer* ***any five full*** *questions.*
All *questions carry* ***equal*** *marks.*

1. a) Using a block diagram explain in detail the steps involved in mathematical modeling of a system. 6

   b) Show that the spring-mass-dashpot system and the LCR circuit are governed by the same differential equations with coefficients being different. 8

2. a) Derive the one-dimensional wave equation for vibrations of a string without damping. 6

   b) Explain the connection between parabolic and hyperbolic equations and the elliptic equation. Further, explain in general terms the nature of the problem and solution of these equations. 8

3. a) Show that the fixed point iterative method has linear convergence. Explain the Aitken Δ^2-process of accelerating the convergence of the linear iterative method. 7

   b) Elaborate on the Sturm sequence method to identify the number of real roots of the algebraic equation $f(x) = 0$. 7

4. a) Prove or disprove that the Newton-Raphson method for finding a simple root of $f(x) = 0$ has quadratic convergence. 7

   b) Find an estimate of the small real root of $\sin x = 1 - x$ by Ramanujan's method. 7

5. a) Explain partial and complete pivoting strategies. Solve the system using the Gauss elimination method.
   $x_1 + x_2 + x_3 = 6$; $3x_1 + 3x_2 + 4x_3 = 20$; $2x_1 + x_2 + 3x_3 = 13$. 8

   b) Solve using the Doolittle method :
   $2x_1 - x_2 + x_3 = -1$; $3x_1 + 3x_2 + 9x_3 = 0$; $3x_1 + 3x_2 + 5x_3 = 4$. 6

P.T.O.

6. a) Develop the Thomas algorithm to solve an $(n \times n)$ tridiagonal system of equations and hence solve $[A/B] = \left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 1 \\ 2 & 3 & 1 & 0 & 2 \\ 0 & 1 & 4 & 2 & 3 \\ 0 & 0 & 1 & 3 & 4 \end{array}\right]$ 9

b) Solve using the Newton-Raphson method $y\cos(xy) + 1 = 0$, $\sin(xy) + x - y = 0$. $(x_0, y_0) = (1, 2)$. (Perform one iteration). 5

7. a) Derive Hermite interpolation formula for the given data $\{x_i, f(x_i), f^1(x_i), i = 0, 1, 2, \ldots, n\}$. 5

b) Show that the Lagrange interpolating polynomial is unique and also find the truncation error involved in Lagrange interpolation. 5

c) Obtain the rational approximation of the form $\frac{a_0 + a_1 x}{1 + b_1 x}$ to e^x. 4

8. a) Establish Gauss-Legendre three point quadrature formula and hence evaluate $\int_{-1}^{1} (1 - x^2)^{3/2} \cos x \, dx$. 5

b) Evaluate $\int_{-1}^{1} \sqrt{1 - x^2} \cos x \, dx$ using Gauss-Chebyshev two and three points formulas. 6